How Moist and Dry Intrusions Control the Local Hydrologic Cycle in Present and Future Climates

Supplemental Materials

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1. LHC Budget Derivation

Applying (2) to (1) and utilizing the linear decomposition $m = M + m_e$ yields

$$\widetilde{\left(\frac{\partial m}{\partial t}\right)} = (\widetilde{E - P}) - [\boldsymbol{\nabla} \cdot \widetilde{(m_e \mathbf{v})}] - [\boldsymbol{\nabla} \cdot \widetilde{(M\mathbf{v})}]. \qquad (A1)$$

We use the following alternate definition of wave activity, borrowed from Huang and Nakamura (2017), which is equivalent to the formulation given in (2) (for proof see Huang and Nakamura 2016),

$$\widetilde{(\ )} = \frac{a}{\cos\phi_e}\int_0^{\Delta\phi} (\ )\cos(\phi_e + \phi')\, d\phi' . \qquad (A2)$$

Here, $\Delta\phi$ is the arc length between the equivalent latitude and the M contour, which can be multi-valued. When it is multi-valued, $\Delta\phi$ will represent the sum of those values (Huang and Nakamura 2016). This definition implies

$$m(t, \lambda, \phi_e + \Delta\phi) \equiv M(\phi_e). \qquad (A3)$$

We start by expanding the lhs of (A1) using (A2),

$$\widetilde{\left(\frac{\partial m}{\partial t}\right)} = \frac{a}{\cos\phi_e}\int_0^{\Delta\phi} \frac{\partial m}{\partial t}\cos(\phi_e + \phi')\, d\phi' . \qquad (A4)$$

Applying Leibniz theorem to (A4),

$$\widetilde{\left(\frac{\partial m}{\partial t}\right)} = \frac{a}{\cos\phi_e}\frac{\partial}{\partial t}\int_0^{\Delta\phi} m\cos(\phi_e + \phi')\, d\phi' - \frac{a}{\cos\phi_e}\frac{\partial\Delta\phi}{\partial t} m\cos(\phi_e + \phi')\Big|_{\phi'=\Delta\phi} . \qquad (A5)$$

Using (A3) yields

$$\widetilde{\left(\frac{\partial m}{\partial t}\right)} = \frac{\partial}{\partial t}(A - M\eta), \qquad (A6)$$

which follows from the definition of LWA and $\eta \equiv a\Delta\phi$.

Now we apply a similar expansion to the second term on the rhs of (A1), expanding the divergence operator in (λ, ϕ') coordinates, which gives

$$[\nabla \cdot \widetilde{(m_e \boldsymbol{v})}] = \frac{a}{\cos \phi_e} \int_0^{\Delta\phi} \left\{ \frac{1}{a \cos \phi_e} \frac{\partial}{\partial \lambda} (m_e u) + \frac{1}{a \cos(\phi_e + \phi')} \frac{\partial}{\partial \phi'} [m_e v \cos(\phi_e + \phi')] \right\} \cos(\phi_e + \phi')\, d\phi', \qquad (A7)$$

where u and v are the zonal and meridional components of $\boldsymbol{v}$, respectively. Using Leibniz theorem again and also the fundamental theorem of calculus,

$$[\nabla \cdot \widetilde{(m_e \boldsymbol{v})}] = \frac{1}{a \cos \phi_e} \frac{\partial}{\partial \lambda} \left[\frac{a}{\cos \phi_e} \int_0^{\Delta\phi} (m_e u) \cos(\phi_e + \phi')\, d\phi' \right] + \frac{a}{\cos \phi_e} \frac{\partial \Delta\phi}{\partial \lambda} (m_e u)|_{\phi'=\Delta\phi} + \frac{1}{\cos \phi_e} [m_e v \cos(\phi_e + \phi')]|_{\phi'=0}^{\phi'=\Delta\phi}. \qquad (A8)$$

(A3) implies $m_e(t, \lambda, \phi_e + \Delta\phi) = m(t, \lambda, \phi_e + \Delta\phi) - M(t, \phi_e) = 0$; combining that with the definition of LWA (A2) gives the middle terms in (6),

$$[\nabla \cdot \widetilde{(m_e \mathbf{v})}] = \frac{1}{a \cos \phi_e} \frac{\partial}{\partial \lambda} (\widetilde{m_e u}) - m_e v. \qquad (A9)$$

The final term in (A1) results from an expansion of the third term on the rhs, based on the fact that $M(t, \phi_e)$ does not depend on (λ, ϕ'). We briefly demonstrate this below, beginning in the same method as (A7).

$$[\nabla \cdot \widetilde{(M \mathbf{v})}] = \frac{a}{\cos \phi_e} \int_0^{\Delta\phi} \left\{ \frac{1}{a \cos \phi_e} \frac{\partial}{\partial \lambda} (M u) + \frac{1}{a \cos(\phi_e + \phi')} \frac{\partial}{\partial \phi'} [M v \cos(\phi_e + \phi')] \right\} \cos(\phi_e + \phi')\, d\phi' \qquad (A10)$$

$$[\nabla \cdot \widetilde{(M \boldsymbol{v})}] = \frac{a}{\cos \phi_e} M \int_0^{\Delta\phi} \left\{ \frac{1}{a \cos \phi_e} \frac{\partial u}{\partial \lambda} + \frac{1}{a \cos(\phi_e + \phi')} \frac{\partial}{\partial \phi'} [v \cos(\phi_e + \phi')] \right\} \cos(\phi_e + \phi')\, d\phi' \qquad (A11)$$

$$[\nabla \cdot \widetilde{(M\boldsymbol{v})}] = M(\widetilde{\nabla \cdot \boldsymbol{v}}) \tag{A12}$$

Applying (A6), (A9), and (A12) to (A1) yields (6).

$$\frac{\partial}{\partial t}(A - M\eta) = (\widetilde{E - P}) - \frac{1}{a\cos\phi_e}\frac{\partial}{\partial \lambda}(\widetilde{m_e u}) + m_e v - M(\widetilde{\boldsymbol{\nabla} \cdot \mathbf{v}})$$

2. Background CWV Gradient Scaling

Since CWV scales exponentially, the sharpening moisture gradient can be explained thermodynamically. Taking the meridional derivative of (9):

$$\frac{\partial M}{\partial y} \sim \left[\frac{\partial \sigma}{\partial y} + \sigma \frac{L_v}{R_{WV}(\widetilde{T_s})^2}\frac{\partial \widetilde{T}_s}{\partial y}\right] M_0 \exp\left(-L_v\left(R_{WV}\widetilde{T_s}\right)^{-1}\right) \tag{B1}$$

We take the ratio between the future state (subscript 2) and present state (subscript 1) as with (10),

$$\frac{\frac{\partial M_2}{\partial y}}{\frac{\partial M_1}{\partial y}} \sim \frac{\left[\frac{\partial \sigma}{\partial y} + \sigma \frac{L_v}{R_{WV}(\widetilde{T_{s,2}})^2}\frac{\partial \widetilde{T}_{s,2}}{\partial y}\right]}{\left[\frac{\partial \sigma}{\partial y} + \sigma \frac{L_v}{R_{WV}(\widetilde{T_{s,1}})^2}\frac{\partial \widetilde{T}_{s,1}}{\partial y}\right]} \exp\left(\alpha \Delta \widetilde{T_s}\right) \tag{B2}$$

The term involving the surface temperature gradient $\frac{\partial \widetilde{T_s}}{\partial y}$ in the brackets should be an order of magnitude larger than the meridional gradient in saturation fraction $\frac{\partial \sigma}{\partial y}$, therefore we neglect $\frac{\partial \sigma}{\partial y}$. Cancelling common remaining terms,

$$\frac{\frac{\partial M_2}{\partial y}}{\frac{\partial M_1}{\partial y}} \sim \frac{\left(\widetilde{T_{s,1}}\right)^2}{\left(\widetilde{T_{s,2}}\right)^2} \frac{\left[\frac{\partial \widetilde{T}_{s,2}}{\partial y}\right]}{\left[\frac{\partial \widetilde{T}_{s,1}}{\partial y}\right]} \exp\left(\alpha \Delta \widetilde{T_s}\right) \tag{B3}$$

We can further simplify this scaling to its primary relationship by splitting $\widetilde{T_{s,2}} = \widetilde{T_{s,1}} + \Delta\tilde{T}_s$ and rewriting $\left(\widetilde{T_{s,1}}\right)^2/\left(\widetilde{T_{s,2}}\right)^2 = \left(1 + \Delta\tilde{T}_s/\widetilde{T_{s,1}}\right)^{-2} \approx \left(1 - 2\Delta\tilde{T}_s/\widetilde{T_{s,1}}\right) \approx 1$ using the binomial approximation (since temperatures are in Kelvin, $\Delta\tilde{T}_s/\left(\widetilde{T_{s,1}}\right) \ll 1$). This gives (11):

$$\frac{\frac{\partial M_2}{\partial y}}{\frac{\partial M_1}{\partial y}} \sim \frac{\frac{\partial \tilde{T}_{s,2}}{\partial y}}{\frac{\partial \widetilde{T_{s,1}}}{\partial y}} \exp\left(\alpha\Delta\tilde{T}_s\right)$$

This completes our argument that the moisture gradient scales with the CC relation and is also thermodynamic, and we conclude by rewriting the above expression without the exponents:

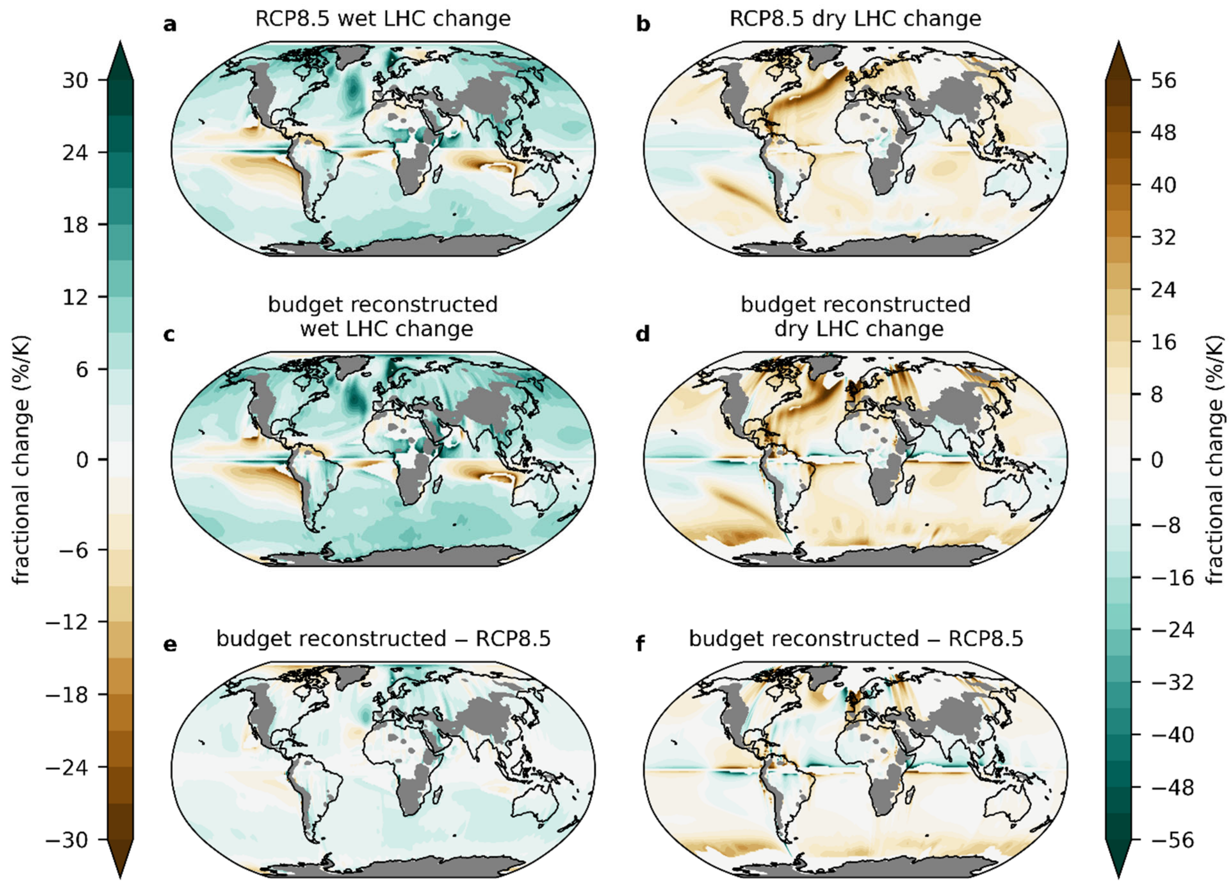
$$\Delta\left(\ln\frac{\partial M}{\partial y}\right) \sim \Delta\left(\ln\frac{\partial \tilde{T}_s}{\partial y}\right) + \alpha\Delta\tilde{T}_s$$

Supplemental References

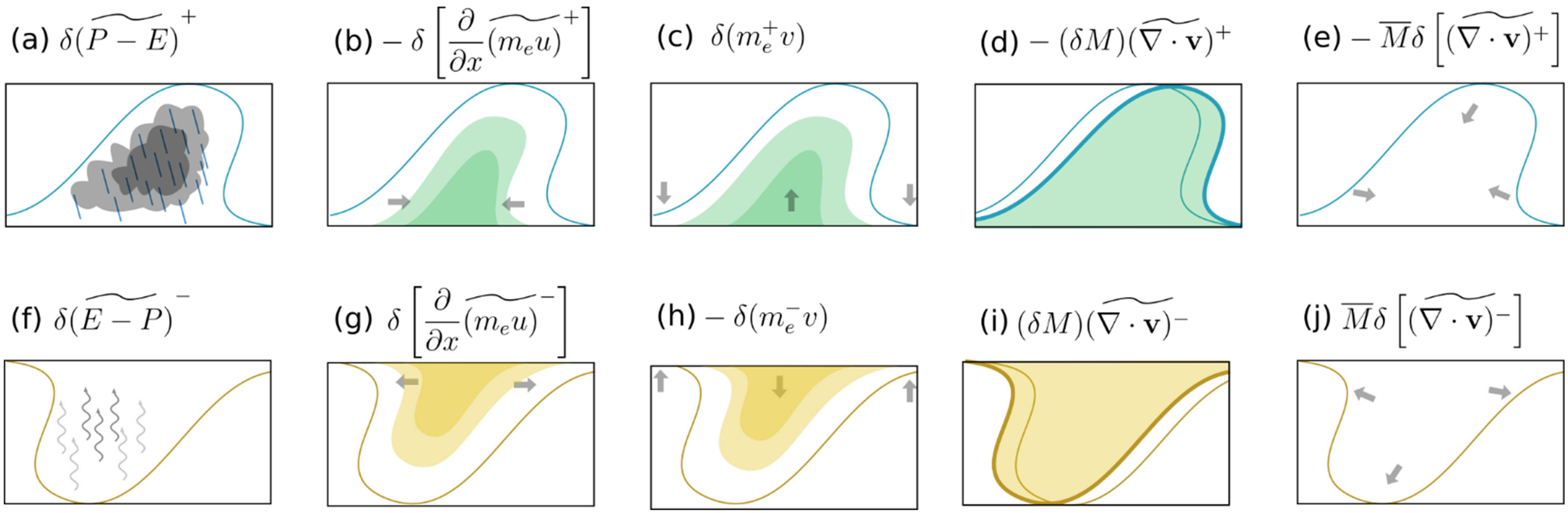
Huang, C. S. Y., and N. Nakamura, 2016: Local Finite-Amplitude Wave Activity as a Diagnostic of Anomalous Weather Events. *Journal of the Atmospheric Sciences*, **73**, 211–229, https://doi.org/10.1175/jas-d-15-0194.1.

——, 2017: Local wave activity budgets of the wintertime Northern Hemisphere: Implication for the Pacific and Atlantic storm tracks. *Geophysical Research Letters*, **44**, 5673–5682, https://doi.org/10.1002/2017gl073760.

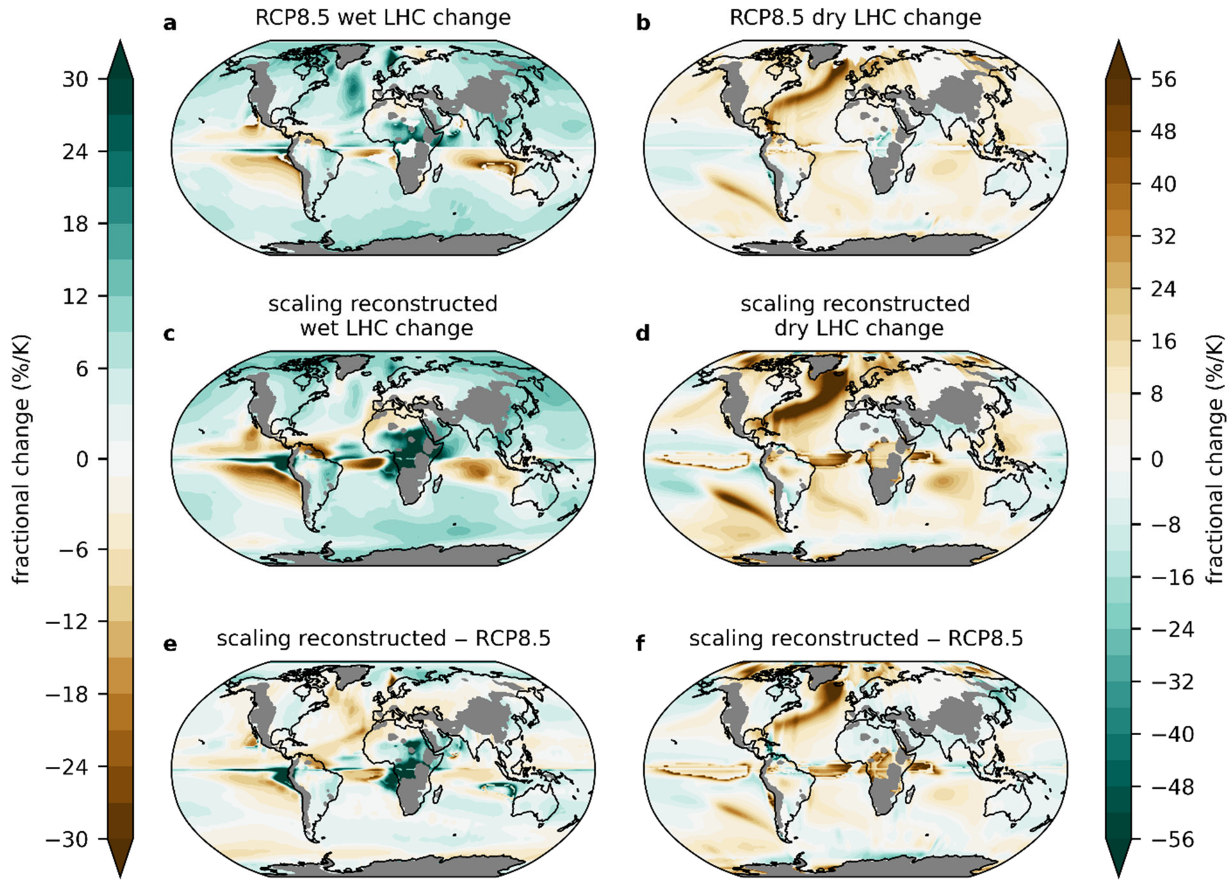
Supplemental Figures



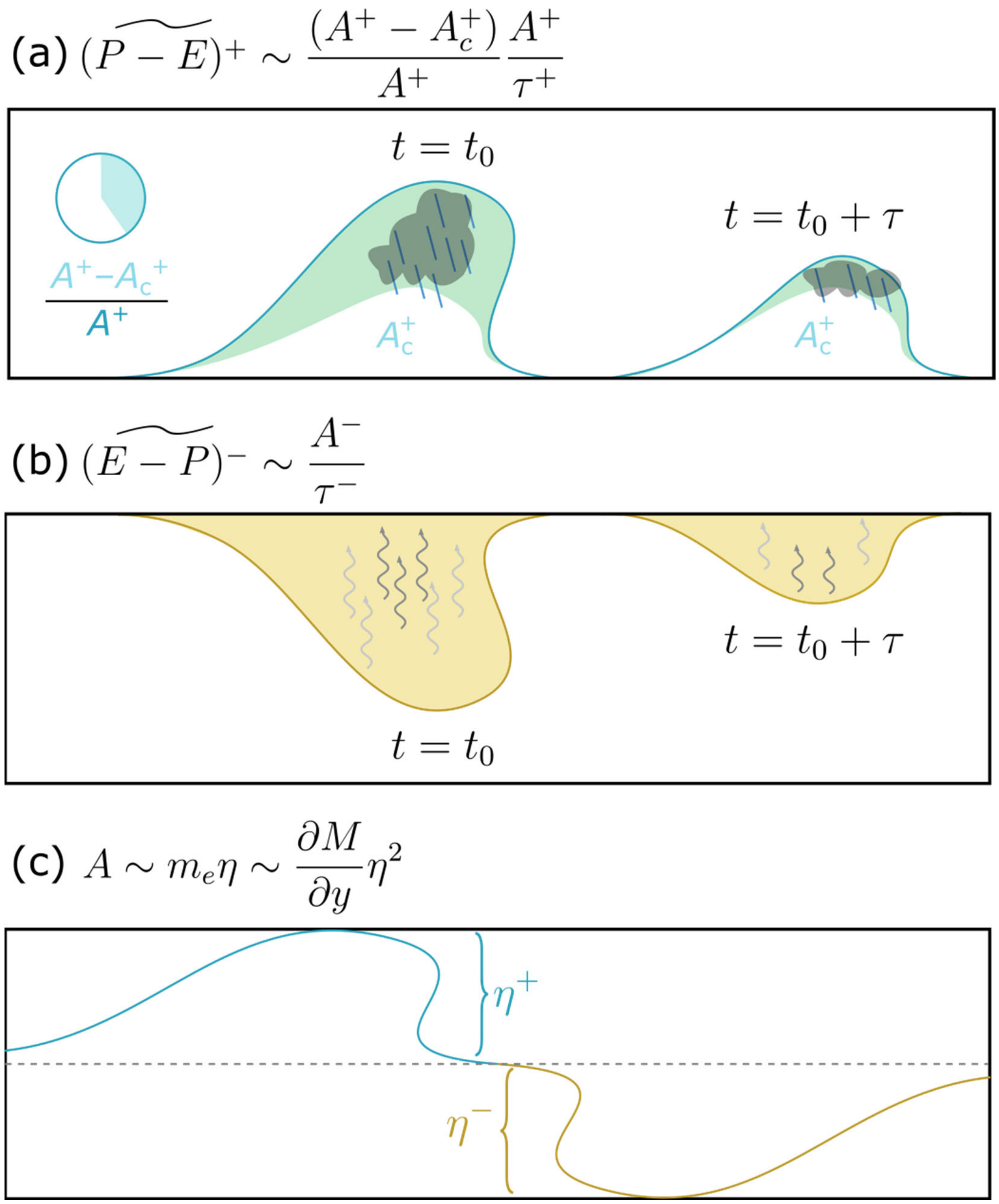
Supplemental Figure 1: LHC budget closure for CESM LENS RCP8.5 scenario [see (8)]. Zonally-symmetric deviations from zero tend to be due to our second assumption, neglecting the change in cross terms between Lagrangian background moisture and low-level convergence. Large regional departures from zero tend to be due to changes in moisture storage [such as the dry bias near the poles in (f)] or effects of the wave activity transform [moist bias in Arctic in (c) and equatorial biases in (f)].



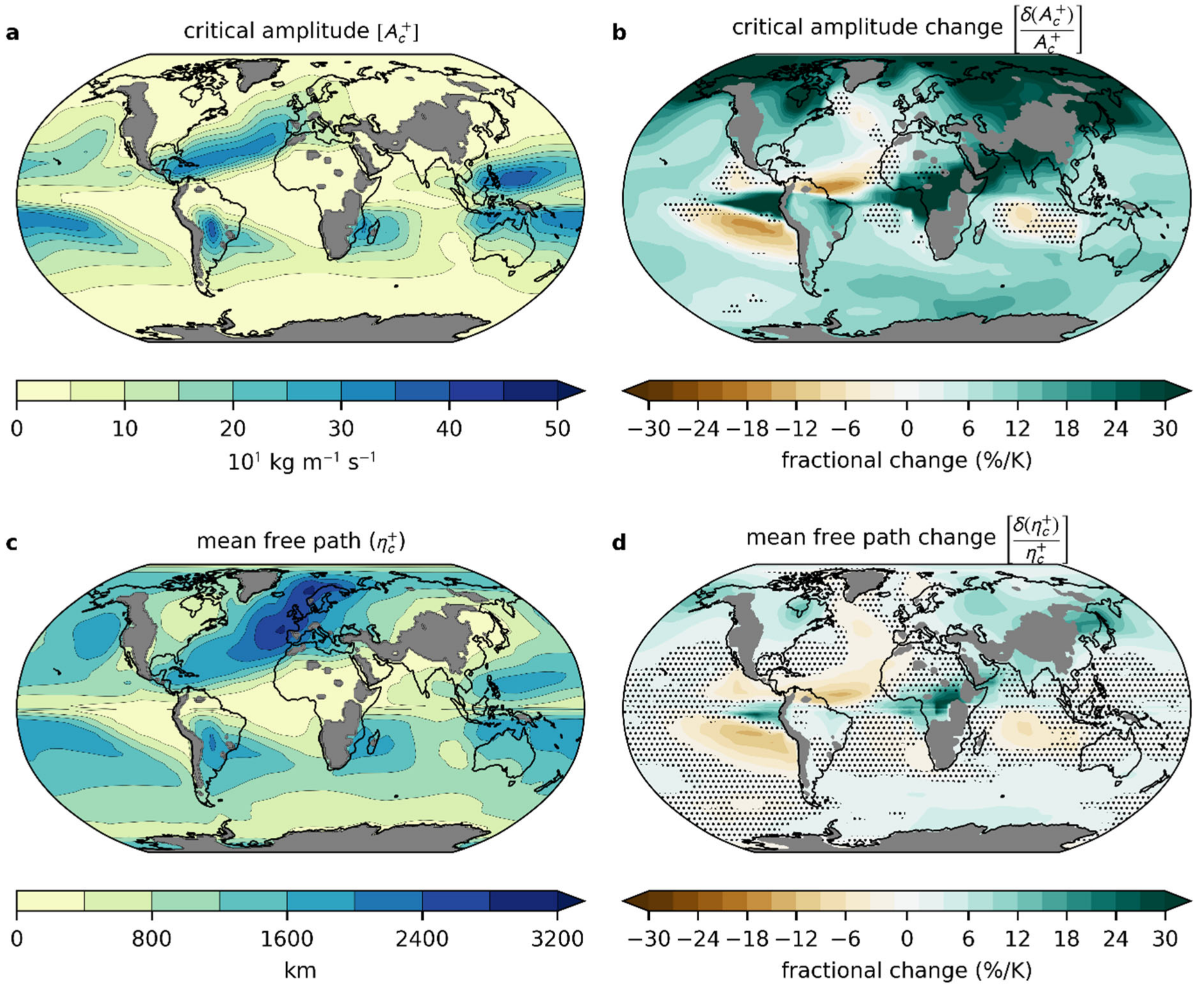
Supplemental Figure 2: Schematic demonstrating the different terms in the LHC budget (7): (a) the wet LHC, which is balanced by (b) zonal moist LWA flux convergence, (c) meridional advection of moist intrusions, (d) increasing background moisture, and (e) low-level convergence. For the dry budget, (f) the dry LHC is balanced by (g) zonal dry LWA flux divergence, (h) meridional advection of dry intrusions, (i) increasing background moisture, and (j) low-level divergence.



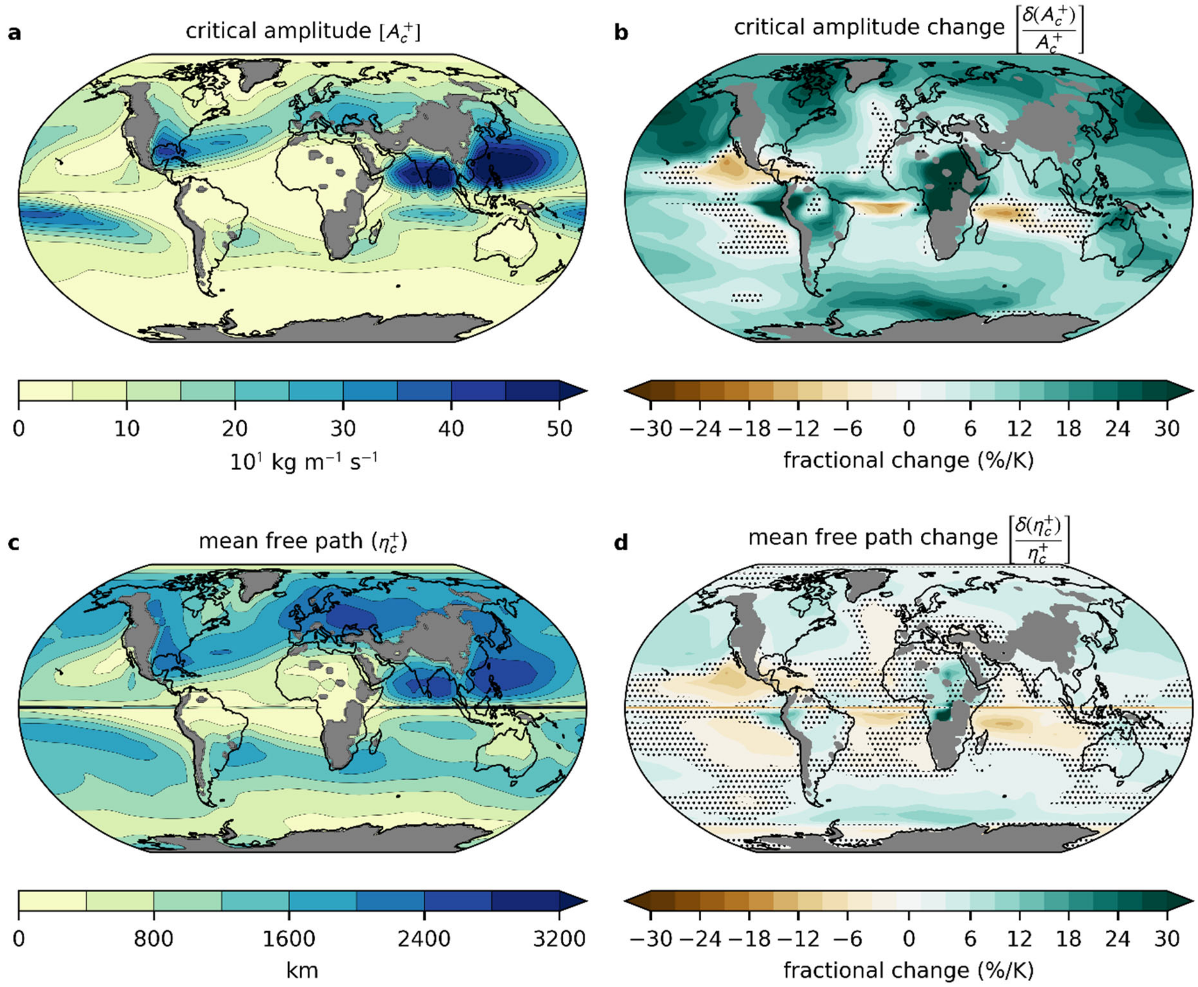
Supplemental Figure 3: Fractional changes in the LHC versus changes reconstructed from the scaling [see (14)] for CESM LENS RCP8.5.



Supplemental Figure 4: Schematic illustrating the different terms in the LHC scaling (13). The wet LHC (a) scales as the product of the proportion of wave activity which participates in the hydrologic cycle, the moist wave activity, and the moist cycling rate, the inverse of the roughly e-folding timescale for moist intrusion damping. The dry LHC (b) scales as the product of the dry wave activity and the dry cycling rate. Moist and dry wave activity (c) can each be scaled as the background moisture gradient times the squared moist or dry length scale, respectively.



Supplemental Figure 5: As in Figure 9, but for December – February mean.



Supplemental Figure 6: As in Figure 9, but for July – August mean.